Sensor Deployment for Source Localization in Wireless Sensor Network System

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Abstract

Source localization is an important task in wireless distributed sensor networking system. One of interest research area is to look for an optimal sensor deployment that can improve the performance of source detection and localization. In this paper, we will present an optimal sensor deployment based on Cramer-Rao Bound analysis for energy based acoustic source localization algorithm. Both theoretical analysis and simulation show that the optimal sensor deployment for this algorithm is to deploy the sensor uniformly and densely in the region where the target may appear. In general, sensors deploying on the straight line should be avoided. Maximum sensor interval is calculated to satisfy the maximum localization estimation error bound under certain confidence level when sensors are uniformly distributed. It has been shown that deploying the sensors densely can get high performance enhancement over price addition.

I. Introduction

Efficient collaborative signal processing algorithms that consume less energy for computation and less communication bandwidth are important in wireless distributed sensor network communication system [1]. An important collaborative signal processing task is source localization.

Existing acoustic source localization methods depend on three types of physical measurements: time delay of arrival (TDOA)[2], direction of arrival (DOA)[3] and source signal strength or power. In practice, DOA measurement typically require costly antenna array on each node. TDOA is suitable for broadband source localization and has been extensively investigated . It requires highly accurate measurement of the relative time delay between sensor nodes. In contrast, received sensor signal strength is comparatively much easier and less costly to obtain from the time series recordings from each sensor.

In [4], a new approach using the acoustic signal intensity (energy measurement) to estimate the source location using *ML* estimation is proposed. In this paper, we will analyze the sensor deployment which can maximize the performance bounds of the energy based source localization based on Cramer Rao Bound analysis. It is shown that the optimal sensor deployment is to deploy sensors uniformly and densely in the sensor field.

CRB is a theoretical lower bound of the variance that we can reach for the unbiased estimation. It is asymptotically achievable for *ML* estimation if the variable is Gaussian distributed. By *Chebyshev*'s inequality, we know that the probability of estimation error is bounded by the ratio of the variance of that random variable and the square of that estimation error. So, the performance bound of source location can be maximized if the *CRB* is minimized. Using this inequality, we predict the maximum sensor interval that can satisfy the required localization error bounds under certain confidence.

The rest of this paper is organized as follows: In section II, we briefly introduce the modelling of acoustic source localization. In section III, we derived the *CRB* for energy based localization problem. Sensor placement to achieve better source location estimation is analyzed and verified by the simulations in section IV. In section V, we calculate the maximum interval that satisfies the estimation error bound under certain confidence level when sensors are uniformly distributed. A conclusion is given in the section VI.

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II. Energy -Based Source Localization

It is known that acoustic energy attenuated at a rate that is inversely proportional to the square of the distance from the source [5]. Based on this knowledge, we may estimate the source location using multiple energy reading at different, known sensor locations.

[4] show that in certain conditions, acoustic energy decay model in the wireless sensor field can be described as the following function:

$$y_i(t) = y_s(t) + \varepsilon_i(t) = g_i \sum_{j=1}^{K} \frac{S_j(t)}{|\boldsymbol{\rho}_j(t) - \mathbf{r}_i|^2} + \varepsilon_i(t) \tag{1}$$

For i=1...N. N is the number of sensors used to estimate the source localization, K is the number of sources, g_i and $\mathbf{r}_i$ are the gain factor and location of the i^{th} sensor, $S_j(t)$ and $\boldsymbol{\rho}_j(t)$ are respectively, the energy emitted by the j^{th} source (measured at 1 meter from the source) and its location during time interval t. $\varepsilon_i(t)$ is a perturbation term that summarizes the net effects of background additive noise and the parameter modelling error.

The probability distribution of $\varepsilon_i(t)$ can be modelled well with an independently, identically distributed Gaussian random variable in practical situations. The mean and variance of each $\varepsilon_i(t)$, denoted by $\mu_i\,(> 0)$ and σ_i^2, can be empirically estimated from the time series data sampled at sensor i using *CFAR* detector [6].

Define $\mathbf{Z} = \begin{bmatrix} \frac{y_1-\mu_1}{\sigma_1} & \frac{y_2-\mu_2}{\sigma_2} & \cdots \frac{y_N-\mu_N}{\sigma_N} \end{bmatrix}^\Gamma$

Equation (1) can be simplified as:

$$\mathbf{Z} = \boldsymbol{GDS}+\boldsymbol{\xi} = \boldsymbol{HS}+\boldsymbol{\xi} \tag{2}$$

Where:

$$\mathbf{G} = diag\begin{bmatrix} \frac{g_1}{\sigma_1} & \frac{g_2}{\sigma_2} & \cdots \frac{g_N}{\sigma_N} \end{bmatrix} \tag{3}$$

$$\mathbf{D} = \begin{bmatrix} \frac{1}{d_{11}^2} & \frac{1}{d_{12}^2} & \cdots & \frac{1}{d_{1K}^2} \\ \frac{1}{d_{21}^2} & \frac{1}{d_{22}^2} & \cdots & \frac{1}{d_{2K}^2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{d_{N1}^2} & \frac{1}{d_{N2}^2} & \cdots & \frac{1}{d_{NK}^2} \end{bmatrix} \tag{4}$$

$$\mathbf{S} = \begin{bmatrix} S_1 & S_2 & \cdots S_K \end{bmatrix}^\Gamma$$

$$\mathbf{H} = \mathbf{GD}$$

$d_{ij} = |\boldsymbol{\rho}_j - \mathbf{r}_i|$ is the Euclidean distance between the i^{th} sensor and the j^{th} source.

The joint distribution function of $\mathbf{Z}$ is:

$$f(\mathbf{Z}|\theta) = (2\pi)^{-\frac{N}{2}} exp\left\{-\frac{1}{2}(\mathbf{Z}-\mathbf{HS})^T(\mathbf{Z}-\mathbf{HS})\right\} \tag{5}$$

The unknown parameters θ in the above function is:

$$\boldsymbol{\theta} = \begin{bmatrix} \boldsymbol{\rho}_1^T & \boldsymbol{\rho}_2^T & \cdots & \boldsymbol{\rho}_k^T & S_1 & S_2 & \cdots & S_k \end{bmatrix}^T$$

Note that we have $K(p+1)$ unknown parameters, where p is the dimension of the location, we need, at least, $K(p+1)$ sensors to localize the K source.

ML estimations with projection solution and *Expectation Maximization* solution can be used to solve this problem [4].

III. Cramer-Rao Bounds for Energy Based Source Localization Problem

CRB is a theoretical lower bound of the variance that we can reach for the unbiased estimation. It is useful to indicate the performance bounds of a particular algorithm. *CRB* also facilitates analysis of factors that impact mostly on the performance of an algorithm. *CRB* is defined as the inverse of the *Fisher Matrix*:

$$\mathbf{J} = -E\left(\frac{\partial}{\partial\boldsymbol{\theta}}\left[\frac{\partial}{\partial\boldsymbol{\theta}}\ln f_{\boldsymbol{\theta}}(\mathbf{Z})\right]\right)$$

For the problem with the joint distribution function described as equation (5), *Fisher matrix* is:

$$\mathbf{J} = \frac{\partial\left(\mathbf{DS}\right)^T}{\partial\boldsymbol{\theta}}\mathbf{G^T}\mathbf{G}\frac{\partial\left(\mathbf{DS}\right)}{\partial\boldsymbol{\theta}^T} \tag{6}$$

Where

$$\frac{\partial\mathbf{DS}}{\partial\boldsymbol{\theta}^T} = \left[\begin{array}{ccccc} \frac{\partial\mathbf{DS}}{\partial\boldsymbol{\rho}_1^T} & \frac{\partial\mathbf{DS}}{\partial\boldsymbol{\rho}_2^T} & \cdots & \frac{\partial\mathbf{DS}}{\partial\boldsymbol{\rho}_K^T} & \frac{\partial\mathbf{DS}}{\partial\mathbf{S}^T}\end{array}\right] \tag{7}$$

$$\frac{\partial\mathbf{DS}}{\partial\mathbf{S}^T} = \mathbf{D} \tag{8}$$

$$\begin{aligned}\mathbf{B}_j &= \frac{\partial(\mathbf{DS})^T}{\partial\boldsymbol{\rho}_j}\\ &= \left[\begin{array}{cccc} \frac{-2S_j}{d_{1j}^3}\mathbf{b}_{1j} & \frac{-2S_j}{d_{2j}^3}\mathbf{b}_{2j} & \cdots & \frac{-2S_j}{d_{Nj}^3}\mathbf{b}_{Nj}\end{array}\right]^T\end{aligned} \tag{9}$$

In above equation, $\mathbf{b}_{ij}$ is the unit vector from source j to sensor i, which can be expressed as:

$$\mathbf{b}_{ij} = \frac{\partial d_{ij}}{\partial\boldsymbol{\rho}_j} = \frac{\boldsymbol{\rho}_j - \mathbf{r}_i}{d_{ij}}$$

Define:

$$\mathbf{B} = \left[\begin{array}{cccc}\mathbf{B}_1 & \mathbf{B}_2 & \cdots & \mathbf{B}_k\end{array}\right] \tag{10}$$

We get the *Fisher Matrix* $\mathbf{J}$ as follows:

$$\mathbf{J} = \left[\begin{array}{c}\mathbf{B^T}\\ \mathbf{D^T}\end{array}\right]\mathbf{G^T}\mathbf{G}\left[\begin{array}{cc}\mathbf{B} & \mathbf{D}\end{array}\right] \tag{11}$$

The CRB is:

$$\mathbf{J}^{-1} = \left(\left[\begin{array}{c}\mathbf{B^T}\\ \mathbf{D^T}\end{array}\right]\mathbf{G^T}\mathbf{G}\left[\begin{array}{cc}\mathbf{B} & \mathbf{D}\end{array}\right]\right)^{-1} \tag{12}$$

The variance of the unknown parameter estimation is bounded by the CRB, i.e.

$$var\left(\widehat{\rho_{i_j}}\right) \geq \left(J^{-1}\right)_{(i-1)p+j,(i-1)p+j}$$

$$\{i = 1\ \cdots\ K,\ \ j = 1\ \cdots\ p\}$$

Where $var\left(\widehat{\rho_{i_j}}\right)$ is the variance of the estimation location for i^{th} source at j^{th} coordinate direction.
For single target, the formula is reduced to:

$$\mathbf{J} = \left[\begin{array}{cc}\mathbf{J}_{11} & \mathbf{J}_{12}\\ \mathbf{J}_{21} & J_{22}\end{array}\right] \tag{13}$$

Where:

$$\mathbf{J}_{11} = \sum_{i=1}^{N}\frac{4S^2 g_i^2}{\sigma_i^2 d_i^6}\mathbf{b}_i\mathbf{b}_i^T \tag{14}$$

$$\mathbf{J}_{21}^T = \mathbf{J}_{12} = -2S \sum_{i=1}^{N} \frac{g_i^2}{\sigma_i^2 d_i^5} \mathbf{b}_i \tag{15}$$

$$J_{22} = \sum_{i=1}^{N} \frac{g_i^2}{\sigma_i^2 d_i^4} \tag{16}$$

$$\mathbf{J}^{-1} = \begin{bmatrix} \mathbf{E}^{-1} & \mathbf{F}\mathbf{L}^{-1} \\ \mathbf{L}^{-T}\mathbf{F}^T & L^{-1} \end{bmatrix} \tag{17}$$

Where:

$$\mathbf{E} = \mathbf{J}_{11} - \frac{1}{\mathbf{J}_{22}} \mathbf{J}_{12} \mathbf{J}_{12}^T$$

$$\mathbf{F} = -\mathbf{J}_{11}^{-1} \mathbf{J}_{12}$$

$$L = J_{22} - \mathbf{J}_{12}^T \mathbf{J}_{11}^{-1} \mathbf{J}_{12}$$

IV. Sensor Placement to Achieve Better Source Location Estimation

By *Chebyshev*'s inequality, we know that the probability of estimation error is less than the ratio of the variance of that random variable and the square of that estimation error, i.e.

$$P\left(\mid X - E\left(X\right) \mid \geq a\right) \leq \frac{Var(X)}{a^2}$$

On the other hand, estimation variance is lower bounded by the CRB. For ML estimation, estimation variance asymptotically approaches its CRB, i.e., the smaller the CRB, the smaller estimate variance we can achieve by ML estimation. And therefore, the less probability of the estimation error we might get. For example, in reality, estimation error less than certain value, say a, is accepted. And we would like the probability of estimation error bigger than a is asymptotically upper bounded by $\frac{CRB}{a^2}$. So, we would like to deploy the sensor in an optimal way such that we can get the smallest *CRB*.

To simplify the problem, let's assume the sensor gain and noise variance is same for each sensor. For single target, we have

$$\mathbf{E} = \kappa \left[\sum_{i=1}^{N} \frac{1}{d_i^6} \mathbf{b}_i \mathbf{b}_i^T - \frac{1}{\sum_{i=1}^{N} \frac{1}{d_i^4}} \sum_{i=1}^{N} \frac{1}{d_i^5} \mathbf{b}_i \sum_{i=1}^{N} \frac{1}{d_i^5} \mathbf{b}_i^T \right] \tag{18}$$

Where

$$\kappa = \frac{4S^2 g^2}{\sigma^2}$$

Define:

$$(\boldsymbol{\rho}_i - \boldsymbol{r}_i) = \begin{bmatrix} \Delta x_i \\ \Delta y_i \end{bmatrix}$$

$$\overline{\Delta y_i} = \frac{d_i^2 \sum_{j=1}^{N} \frac{\Delta y_j}{d_j^6}}{\sum_{j=1}^{N} \frac{1}{d_j^4}} \tag{19}$$

$$\overline{\Delta x_i} = \frac{d_i^2 \sum_{j=1}^{N} \frac{\Delta x_j}{d_j^6}}{\sum_{j=1}^{N} \frac{1}{d_j^4}} \tag{20}$$

$$\mathbf{L_Y} = \begin{bmatrix} \frac{\Delta y_1 - \overline{\Delta y_1}}{d_1^4} & \frac{\Delta y_2 - \overline{\Delta y_2}}{d_2^4} & \cdots & \frac{\Delta y_N - \overline{\Delta y_N}}{d_N^4} \end{bmatrix}$$

$$\mathbf{L_X} = \begin{bmatrix} \frac{\Delta x_1 - \overline{\Delta x_1}}{d_1^4} & \frac{\Delta x_2 - \overline{\Delta x_2}}{d_2^4} & \cdots & \frac{\Delta x_N - \overline{\Delta x_N}}{d_N^4} \end{bmatrix}$$

$$\theta = \mathbf{L_X} \wedge \mathbf{L_Y}$$

We have:

$$\mathbf{E} = \kappa \begin{bmatrix} e_{11} & e_{12} \\ e_{12} & e_{22} \end{bmatrix} \tag{21}$$

Where:

$$e_{11} = |\mathbf{L_X}|^2$$

$$e_{12} = < \mathbf{L_X}, \mathbf{L_Y} > = |\mathbf{L_X}||\mathbf{L_Y}|cos\theta$$

$$e_{22} = |\mathbf{L_Y}|^2$$

Then:

$$CRB(\hat{x}) = \mathbf{E}_{11}^{-1} = \frac{1}{\kappa}\frac{1}{|\mathbf{L_X}|^2(1 - cos^2\theta)} \tag{22}$$

$$CRB(\hat{y}) = \mathbf{E}_{22}^{-1} = \frac{1}{\kappa}\frac{1}{|\mathbf{L_Y}|^2(1 - cos^2\theta)} \tag{23}$$

$$|\mathbf{L_X}| = \left\{ \sum_{i=1}^{N} \frac{(\Delta x_i - \overline{\Delta x_i})^2}{(\Delta x_i + \Delta y_i)^8} \right\}^{1/2} \tag{24}$$

$$|\mathbf{L_Y}| = \left\{ \sum_{i=1}^{N} \frac{(\Delta y_i - \overline{\Delta y_i})^2}{(\Delta x_i + \Delta y_i)^8} \right\}^{1/2} \tag{25}$$

From equation (22) and equation (23), we know that $CRB(\hat{x}) \to \infty$ or $CRB(\hat{y}) \to \infty$ when either of the following situation occurs: (i) when $\kappa \to 0$, (ii) when $|\mathbf{L_X}| \to 0$ or $|\mathbf{L_Y}| \to 0$; and (iii) when $\theta \to 0$. (i) happens only if $S \to 0$. This will never happen as the localization is triggered only if there is target detection. (iii) may occur when $\mathbf{L_X} \propto \mathbf{L_Y}$. Following we will discuss the sensor placement for condition (ii) and (iii).

From (24) and (25), we know that $|\mathbf{L_X}|$ or $|\mathbf{L_Y}|$ goes to zero if, (1) all sensors are far from the source since they are inverse to the sixth order of the distance between sensor and the source, or, (2) $\Delta x_i - \overline{\Delta x_i}$ goes to zero for all i.

When there is only one sensor close to the target, we can show that both $\mathbf{L_Y}$ and $\mathbf{L_X}$ become zero vector, i.e., $|\mathbf{L_X}|$ or $|\mathbf{L_Y}|$ goes to zero.

When there are only two sensors close to the target,

$$\mathbf{L_Y} = \frac{1}{d_1^4} \begin{bmatrix} \alpha^4(\Delta\rho_{y_1} - \alpha^2 \Delta\rho_{y_2}) & \Delta\rho_{y_2} - \frac{1}{\alpha^2}\Delta\rho_{y_1} & 0 & \cdots & 0 \end{bmatrix}$$

$$\mathbf{L_X} = \frac{1}{d_1^4} \begin{bmatrix} \alpha^4(\Delta\rho_{x_1} - \alpha^2 \Delta\rho_{x_2}) & \Delta\rho_{x_2} - \frac{1}{\alpha^2}\Delta\rho_{x_1} & 0 & \cdots & 0 \end{bmatrix}$$

Where we assume that sensor 1 and sensor 2 are close to the target and all other sensors are far away from the target and $\alpha = \frac{d_1}{d_2}$. When $\alpha \to 1$, i.e., target sits between the middle of the nearest two sensors and others are far from the target, L_x and L_y becomes parallel, $CRB \to \infty$. Other possible sensor placement that may cause $\mathbf{L_X}$ and $\mathbf{L_Y}$ becomes parallel is that all sensors are placed on the same line and the target is also on that line.

To achieve better performance, we would like to get the smaller *CRB*. Therefore, we need to maximize $|\mathbf{L_Y}|$, $|\mathbf{L_X}|$ and set $\theta = 90°$.

When $|\mathbf{L_Y}|$ and $|\mathbf{L_X}|$ are fixed, we get the minimum CRB if $\theta = 90°$, i.e., $\mathbf{L_X}$ is orthogonal to $\mathbf{L_Y}$. So, we need $< \mathbf{L_X}, \mathbf{L_Y} >= 0$, i.e.

$$\sum_{i=1}^{N} \frac{\Delta x_i \Delta y_i}{d_i^8} - \frac{1}{\sum_{j=1}^{N} \frac{1}{d_j^4}} \sum_{i=1}^{N} \frac{\Delta y_i}{d_i^6} \sum_{i=1}^{N} \frac{\Delta x_i}{d_i^6} = 0 \tag{26}$$

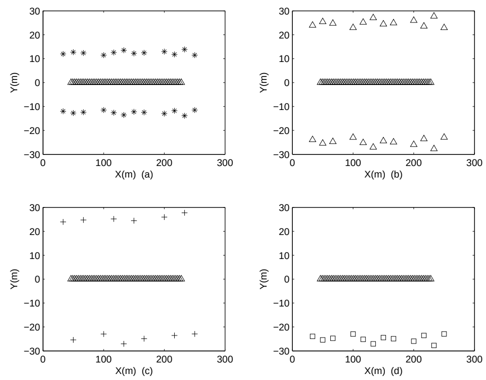

Fig. 1. Sensor Deployment, (a) dense sensor close to road, (b) dense sensor, (c) loose sensor located at two side, (d) loose sensor located at one side

One way to satisfy (26) is to deploy the sensor around the source symmetrically both in X coordinates and Y coordinates. So, the number of sensor we need is: K=4k, where k is integer.

When target is moving, it is hardly to get $\mathbf{L_X} \perp \mathbf{L_Y}$ at each point in the region. However, we can deploy the sensor uniformly and densely in the region so that at each point in the region, we can get the approximate orthogonal condition of $\mathbf{L_X}$ and $\mathbf{L_Y}$. (Suppose the region is big enough, and we don't consider the boundary of the region). When we know that the target is on the road, (26) is satisfied when the sensors are deployed symmetrically at the two side of the road.

Another way to decrease the CRB is to increase $|\mathbf{L_Y}|$ and $|\mathbf{L_X}|$. (25) and (24) show that $|\mathbf{L_Y}|^2$ and $|\mathbf{L_X}|^2$ relates to the inverse of the sixth order of d_i. The smaller d_i, the bigger of $|\mathbf{L_Y}|^2$ and $|\mathbf{L_X}|^2$ we can get. To get the smaller d_i, we can deploy sensors densely in the region and use the sensor which have bigger received energy to estimate the localization[1]. When the target is on the road, we can deploy the sensor closely to the road.

From above analysis, we know that for single target source localization, the optimal sensor deployment that minimizing CRB is to deploy the sensor uniformly and densely in the region. When the target is constraint on the road, we can deploy the sensors symmetrically, densely and close to the road.

Simulations have been conducted to verify the theoretical analysis conclusion for the optimal sensor placement that minimizing the *CRB*. Four different sensor deployment are used to evaluate the CRB. Simulation results are shown in Fig. 2 with the sensor deployment shown as Fig. 1. The results show that case 1 has smaller CRB than that of case 2. This is because the sensor is closer to the road for case 1 than that for case 2. Case 3 has bigger CRB than that of case 2 because the sensors are looser for case 3 than that for case 2 with other conditions are same. Case 4 has bigger CRB than that of case 3 because for case 3, sensors are symmetrically deployed at the both side of the road while in case 4, sensors are deployed at one side. These simulation results are consistent with our theoretically analysis.

V. Maximum Interval Between Sensors When Sensors Are Uniformly Distributed

Now let's assume sensors are uniformly distributed, we would like to calculate the maximum interval between sensors when certain fault tolerance (confidence) are required.

Suppose sensors are deployed as Fig. 3, we pick the sensors that have the highest $4k$ received energy to estimate the target location. Since sensors are symmetric around the target, from equation (19) and equation (20), we know that $\overline{\Delta y_i}$ and $\overline{\Delta x_i}$ are zero for all i. Therefore, $\mathbf{L_X}$ and $\mathbf{L_Y}$ are reduced to:

$$\mathbf{L_Y} = \begin{bmatrix} \frac{\Delta y_1}{d_1^4} & \frac{\Delta y_2}{d_2^4} & \cdots & \frac{\Delta y_N}{d_N^4} \end{bmatrix}$$

$$\mathbf{L_X} = \begin{bmatrix} \frac{\Delta x_1}{d_1^4} & \frac{\Delta x_2}{d_2^4} & \cdots & \frac{\Delta x_N}{d_N^4} \end{bmatrix}$$

[1]By energy decay model, the sensors that receive the bigger energy are closer to the target, and therefore, have smaller d_i

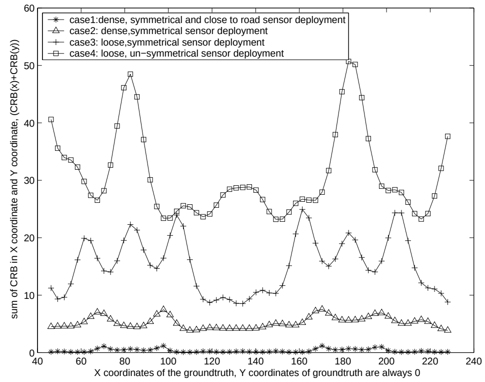

Fig. 2. CRB for different sensor deployment shown as Fig. 1

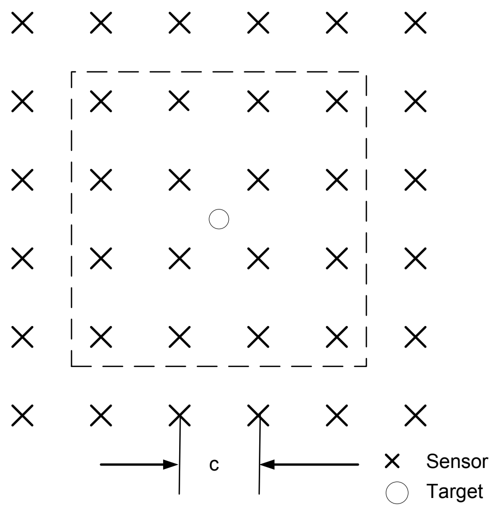

Fig. 3. Uniformly distributed sensor field

In such case, X coordinate and Y coordinate are equivalent. In addition, for every estimation, the relative distance matrix from the sensor to the target can be assumed to be same. So, $CRB(x)$ and $CRB(y)$ are equal and keep constant for every estimation point. We would like

$$\begin{aligned} E[P\left(\left|\hat{\rho_x} - E\left(\rho_x\right)\right| \geq a\right)] &\leq E[\frac{\mathit{Var}(\rho_x)}{a^2}] \\ &= \frac{\mathit{CRB(x)}}{a^2} \leq (1-\eta_\alpha) \end{aligned} \tag{27}$$

Where η_α is the confidence level. Insert equation (22) into (27) and note that θ is 90° for the uniformly distributed sensors, we have:

$$|\mathbf{L_X}|^2 \geq \frac{1}{\kappa(1-\eta_\alpha)a^2} = \frac{1}{4g^2(1-\eta_\alpha)a^2\mathit{SNR}} \tag{28}$$

Where:

$$|\mathbf{L_X}|^2 = \sum_{i=1}^{4k} \frac{(\Delta x_i)^2}{(\Delta x_i + \Delta y_i)^8} \tag{29}$$

$$SNR = \frac{S^2}{\sigma^2}$$

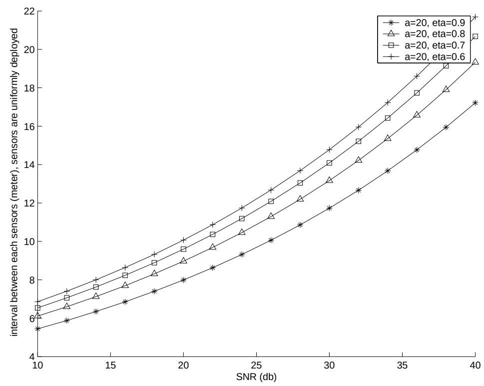

Fig. 4. relation between maximum sensor interval and SNR, the confidence level

Define:

$$\Delta x_i' = \frac{\Delta x_i}{c}$$

$$\Delta y_i' = \frac{\Delta y_i}{c}$$

Then:

$$\mathbf{L}'_{\mathbf{X}} = \left[\begin{array}{cccc} \frac{\Delta x_1'}{{d_1'}^4} & \frac{\Delta x_2'}{{d_2'}^4} & \cdots & \frac{\Delta x'_N}{{d'_N}^4} \end{array} \right]$$

$$|\mathbf{L}'_{\mathbf{X}}|^2 = \sum_{i=1}^{N} \frac{(\Delta x_i')^2}{(\Delta x_i' + \Delta y_i')^8} = \mathbf{L}_{\mathbf{X}} c^6$$

where c is the maximum interval between the neighboring sensors. Then:

$$c \leq \{4g^2(1-\eta_\alpha)a^2|\mathbf{L}'_{\mathbf{X}}|^2 SNR\}^{1/6} \tag{30}$$

If we choose k=4, i.e., we use 16 sensors denoted as Fig. 3 to estimate the target, then

$$\begin{aligned} \{\mathbf{L}'_X\}_{1\sim4} = \{\mathbf{L}'_X\}_{5\sim8} = \{\mathbf{L}'_X\}_{9\sim12} = \{\mathbf{L}'_X\}_{13\sim16} \\ = \left[\begin{array}{cccc} -1.5 & -0.5 & 0.5 & 1.5 \end{array} \right] \end{aligned}$$

Fig. 4 and Fig. 5 show the relation of maximum sensor interval, *SNR*, estimation error bound (a) and the confidence level (η_α) in the uniformly distributed sensor network. It shows that maximum sensor interval is larger when SNR in the sensor network is larger, i.e., if the SNR is larger, we can deploy the sensor looser while we can still satisfy the accepted estimation error bound (a) under certain confidence level η_α. We also notice that for fixed *SNR* and accepted estimation error bound a, confidence level increase a lot when we deploy the sensor a little bit denser. This is because the sixth order relationship between the confidence level and maximum sensor interval as denoted in equation (30). For example, fix a and *SNR*, then $\frac{1-\eta_{\alpha_1}}{1-\eta_{\alpha_2}} = \{\frac{c_1}{c_2}\}^6$, fix η_α and *SNR*, then $\frac{a_1}{a_2} = \{\frac{c_1}{c_2}\}^3$. Therefore, performance enhancement over price addition (we need more sensors for dense sensor field) is high.

VI. CONCLUSION

CRB has been derived for energy based source localization problem. Based on the *CRB* analysis, we conclude that the optimal sensor deployment for better performance of energy based acoustic source localization is:

1) Use dense sensor
2) Uniformly distributed in the region

When the target is constraint on the road, we can deploy the sensor symmetrical and densely and close to the road. The limit of such case is to deploy the sensor uniformly and densely on the road. However, such

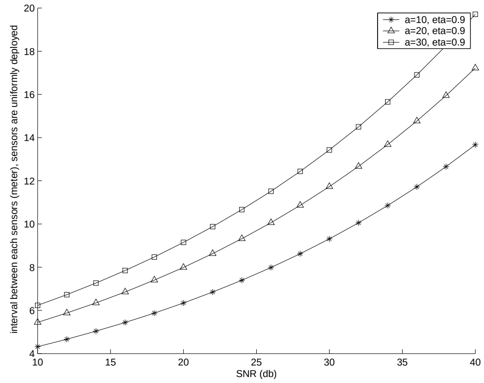

Fig. 5. relation between maximum sensor interval and SNR, the estimation error bounds under certain confidence level

situation should be avoided as it meets the condition that all sensors are on the line with the target, which will abrupt the CRB.

In general, we need at least two sensors close to the target to avoid the infinite of *CRB*. When only two sensors are close to the target, the *CRB* can also goes to infinite when the target sits on the middle of the two closest sensors (Assume others are far away from the target). To avoid this possibility, we need three sensors close to the target. Deploying sensors on the straight line should also be avoided as it can cause the *CRB* goes to infinite when the source is on that line.

Maximum sensor interval is calculated to satisfy the estimation error bound under certain confidence level when sensors are uniformly distributed. Sensors can be deployed looser in high *SNR* environment. Overall, we proved that deploying the sensors densely can get much high performance enhancement over price addition.

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